Modeling and Measurement of MIMO Relay Channels

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***Abstract*—This paper proposes a relay channel model which takes into account the influence of relay location and antenna configuration. Simulation results show that when relay is in the line-of-sight (LOS) propagation environment, the channel capacity will degrade. To improve the performance of the channel, water-filling (WF) algorithm is used to allocate the power, simulation results shows that the WF-based relay channel outperforms the uniform power allocation channel. The radiation pattern of the relay antenna is also an importance factor which affects the channel capacity in this case. The widely used Rayleigh channel model overestimates the performance of the channel compared with our model when the location of the relay is not in the scenario where Ricean K-factor is zero. Measurements have been done at 2.53 GHz to investigate the above-rooftop and ground-level relay channels. Results indicate that above-rooftop deployment is very efficient to improve the capacity and coverage.**

***Index Terms*—Relays, MIMO systems, Propagation, channel capacity, Rayleigh channels.**

I. INTRODUCTION

Relays are often considered as a means to improve the performance of infrastructure based network by increasing their coverage area and exploiting the spatial diversity. The existing literatures mainly focus on studying the various methodologies to obtain better channel performance. However, little attention is paid to the channel modeling for relays. The widely used channel model for the relay system is the Rayleigh channel model [1], [2], [3], which holds in the case where relay lies in the non-line-of-sight (NLOS) scenario of both the source and the destination. The channels between the source and relay, and the relay and destination are considered as the full rank multiple-input multiple-output (MIMO) channels. The effect of the location of the relay on the system performance is usually ignored in the analysis. [4] investigated the power and location optimization for decode-and-forward relay network, but the fading coefficient of the channel is still assumed to be distributed according to Gaussian distribution. However, to maximize the coverage, the relay is typically placed in the LOS scenario of the source [2]. Therefore, the Rayleigh channel model is no longer suitable to analyze practical relay channels. In addition, since there is LOS component propagating from the source to relay, in the case of the MIMO communication, the channel between the relay and the source is correlated. Hence the capacity performance of the whole channel will degrade, and it is necessary to use techniques such as cross-polarized antenna or optimal power allocation to improve the performance in the channel.

In this paper, we focus on the channel modeling for a simple two-hop single relay system.The important parameters which affect the channel performance will be analyzed. Cross-polarized antennas arranged in a uniform linear array (ULA) are used in the simulation. The location of the relay and the configuration of the relay antenna will affect the performance of the channel and it is necessary to take these effect into account when planning a relay network. In order to validate our assumptions, measurements have been done at 2.53 GHz to investigate the above-rooftop and ground-level relay channels. Water-filling power allocation strategy is employed at the source to exploit the full capacity of the relay channel.

The paper is organized as follows. In section II, the relay channel model is formulated, and the influence of the propagation environment is taken into account. Section III discusses the measurement results and analyzes the factors that affect the capacity of the relay channel. The water-filling algorithm is employed in Section IV and concluding remarks can be found in Section V.

II. SYSTEM MODEL

A. Two-Hop Single Relay System

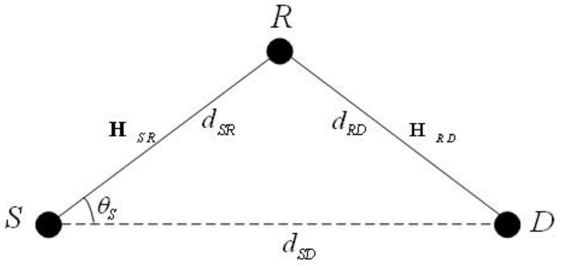

Fig. 1. Two-hop relay channel.

The basic relay system model is shown in Fig.1. We consider a two-hop single relay channel with multiple antenna elements at the source, relay and destination. The relay simply amplifies and forwards the received signal to the destination. The source and destination are marked as S and D in the figure, while R denotes the relay. We ignore the link between

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the source and the destination. The received signal at the destination can be expressed as below [1], [2]

$$\mathbf{y} = \mathbf{H}_{RD}\mathbf{G}\mathbf{H}_{SR}\mathbf{x} + \left[\begin{array}{cc} \mathbf{H}_{RD}\mathbf{G} & \mathbf{I} \end{array} \right] \left[\begin{array}{c} \mathbf{n}_{SR} \\ \mathbf{n}_{RD} \end{array} \right] \quad (1)$$

where $\mathbf{x}$ and $\mathbf{y}$ are transmitted and received signal vectors. $\mathbf{n}_{SR}$ and $\mathbf{n}_{RD}$ are noise vectors at the relay and destination, respectively, which have zero mean and identity covariance matrices. $\mathbf{G}$ contains the power amplification factors at the relay.

In the existing literature, $\mathbf{H}_{SR}$ and $\mathbf{H}_{RD}$ are normally assumed to be full rank Rayleigh matrices, which entries are identically independent distributed (i.i.d.) complex Gaussian random variables with unit variance. In practical propagation environments, these assumptions may not hold true. In the following part, we will propose a relay channel model which can take into account both LOS and NLOS propagation environments, as well as cross-polarized antennas.

B. Relay Channel Model

It is reported in [5], [6], [7] that the general MIMO channel can be modeled as the sum of weighed LOS and NLOS components. Therefore, for a more general case, we take into account the influence of the propagation environment. The channel matrices $\mathbf{H}_{SR}$ and $\mathbf{H}_{RD}$ in (1) can now be written as

$$\begin{aligned} \mathbf{H}_{SR} &= \sqrt{P_{SR}}\widetilde{\mathbf{H}}_{SR} \\ &= \sqrt{P_{SR}}\left(\sqrt{\frac{K_{SR}}{K_{SR}+1}}\widetilde{\mathbf{H}}_{SR}^{LOS} + \sqrt{\frac{1}{K_{SR}+1}}\widetilde{\mathbf{H}}_{SR}^{NLOS}\right) \end{aligned} \quad (2)$$

$$\begin{aligned} \mathbf{H}_{RD} &= \sqrt{P_{RD}}\widetilde{\mathbf{H}}_{RD} \\ &= \sqrt{P_{RD}}\left(\sqrt{\frac{K_{RD}}{K_{RD}+1}}\widetilde{\mathbf{H}}_{RD}^{LOS} + \sqrt{\frac{1}{K_{RD}+1}}\widetilde{\mathbf{H}}_{RD}^{NLOS}\right) \end{aligned} \quad (3)$$

where all the matrices in (2) and (3) with a tilde "$\sim$" on the head denote the normalized channel matrices. $\widetilde{\mathbf{H}}_{SR}^{LOS}$ and $\widetilde{\mathbf{H}}_{RD}^{LOS}$ represent the contribution of the LOS components in the source-relay MIMO channel and relay-destination MIMO channel, respectively. These matrices are determined by the specific distance between the transmitter and the receiver and the configurations of the antennas. $\widetilde{\mathbf{H}}_{SR}^{NLOS}$ and $\widetilde{\mathbf{H}}_{RD}^{NLOS}$ take into account the influence of the scattering components during the propagation. For simplicity, they are modeled as Rayleigh matrices, which entries are zero-mean unit-variance complex Gaussian random variables. K_{SR} and K_{RD} are Ricean K-factors for source-relay and relay-destination scenarios, which can be modeled as a function of the distance between the transmitter and receiver [8], [7], [9]. In a LOS scenario, the value of K-factor can be very high, as K decreases, the contribution of scattered components to the channel becomes more significant while the effect of the LOS component becomes less important, until K reduces to zero, there will be no LOS component and it is an absolute NLOS scenario. P_{SR} and P_{RD} are the received power at the relay and the destination, respectively. The path loss is approximated as $1/d^{\gamma}$, where d is the distance between the transmitter and the receiver, and γ is the path loss exponent depending on the propagation environment.

The cross-polarized MIMO channel for LOS scenario can be modeled as [10]

$$\mathbf{H}_{LOS} = \left[\begin{array}{cc} \mathbf{H}^{'} \odot \mathbf{A}^{VV} & \mathbf{H}^{'} \odot \mathbf{A}^{VH} \\ \mathbf{H}^{'} \odot \mathbf{A}^{HV} & \mathbf{H}^{'} \odot \mathbf{A}^{HH} \end{array} \right] \quad (4)$$

where $\odot$ denotes the element wise multiplication. $\mathbf{H}'$ is the LOS channel matrix for MIMO system with co-polarized antennas. $\mathbf{A}$ is the matrix taking into account the polarization mismatch. When transmit and receive antennas are all strictly aligned, $\mathbf{A}^{VV}$ and $\mathbf{A}^{HH}$ are all-one matrices, while $\mathbf{A}^{HV}$ and $\mathbf{A}^{VH}$ are all-zero matrices. In realistic environments, transmitter and receiver are usually not in such ideal situation, we assume that the transmit elements are rotated at an angle θ_p compared with the receive antenna. Then the elements of $\mathbf{A}^{VV}$ and $\mathbf{A}^{HH}$ become $\cos\theta_p$, and the elements of $\mathbf{A}^{VH}$ and $\mathbf{A}^{HV}$ become $\mp\cos(\pi/2-\theta_p)$. This angle is denoted as the polarization rotation angle.

In addition, there will be a loss for the LOS channel because of the orientations of the antenna arrays. The boresights of the transmit and receive antennas do not strictly point to other. In this case, the misplacement angles are actually the angle of departure (AOD) at the transmitter and the angle of arrival (AOA) at the receiver for LOS components. Since the distance between the transmitter and the receiver is usually very large compared with the antenna size, it is reasonable to assume that the AODs or AOAs for the all elements on the same array are approximately the same.

The channel matrices for NLOS scenario can be simply assumed as Rayleigh matrices despite using polarized antennas, for the reason stated in [10]. In a rich scattering environment, the number of scatterers is very large. According to the central limit theorem, the elements of $\mathbf{H}_{NLOS}$ can be approximated as Gaussian variables, which is in accordance with the well-known classical i.i.d. Gaussian model.

C. Relay Amplification Factor

In a non-regenerative relay channel, the relay simply amplifies and forwards the received signal in the second transmit time slot. To ensure that the total transmitted power in two time slots is restricted to P_0, the amplification factor at the relay is selected as

$$g = \sqrt{\frac{P_0}{\frac{P_0}{2N_S}\|\mathbf{H}_{SR}\|_F^2 + 2N_R\sigma_n^2}} \quad (5)$$

where g is the diagonal element of the relay amplification matrix $\mathbf{G}$, and $\|\cdot\|_F$ denotes the Frobenius-norm. P_0 is the total transmitted power at the source. In our model, the power is assumed to be allocated uniformly over all the antennas. The entries of the noise vectors $\mathbf{n}_{SR}$ and $\mathbf{n}_{RD}$ are assumed to be i.i.d. complex Gaussian variables with equal variance σ_n^2.

D. Channel Capacity

The capacity of the relay channel can be obtained as below

$$C = 0.5\log_2 \det\left(\mathbf{I} + \frac{P_0}{2N_S\sigma_n^2}\mathbf{H}_{RD}\mathbf{G}\mathbf{H}_{SR}\mathbf{H}_{SR}^H\mathbf{G}^H\mathbf{H}_{RD}^H \times \left(\mathbf{I} + \mathbf{H}_{RD}\mathbf{G}\mathbf{G}^H\mathbf{H}_{RD}^H\right)^{-1}\right) \tag{6}$$

where $(\cdot)^H$ denotes the conjugate transpose of the matrix. The factor 0.5 in the capacity formula comes from the fact that, the signal is actually transmitted in two time slots so that the efficiency drops by a half when the units are in bits per second.

III. Results and Discussion

A. Measurement results

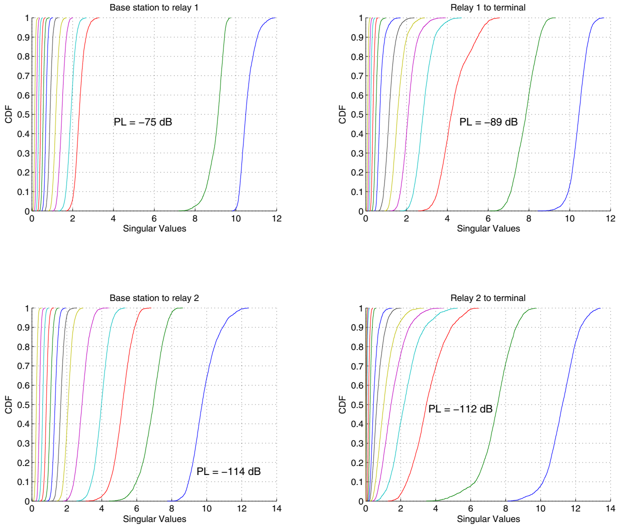

Fig. 2. The singular values and the path loss (PL) for source-relay and relay-destination channels.

We made a measurement on Technical University Berlin (TUB) campus at 2.53 GHz with two different relay positions. One is fixed on the rooftop of the Math building (relay 1), and the other is at the ground level (relay 2) about the same hight as the terminal. Cross-polarized patch antennas are used at the base station, relay and terminal. The comparisons of the singular values of the normalized channel is plotted in Fig. 2. Measurement results show that the position of the relay affects the properties of the channel. The rank of the relay 1 channel is much smaller than that of relay 2 channel, since relay 1 is in the LOS scenario of the base station. The two significant singular values in the top left sub-figure are caused by the cross-polarized antennas. On the other hand, the signal to noise ratio (SNR) is much better for both links of relay 1 on the rooftop. The path loss is indicated in the sub-figures. In particular, the links of relay 2 channel are very weak, and less suitable for data transmission for realistic transmit power.

The capacity of the relay channels are illustrated in Fig. 3. From the figure we can see that the capacity of relay 1 channel is much higher than that of relay 2 channel. This is for the reason that the SNRs for source-relay 2 and relay 2-destination links are very low, compared with the SNRs in relay 1 case. The extreme high capacity is caused by the high SNRs in relay 1 channels. However, in realistic system, the capacity may be smaller because of the impairments at the receiver.

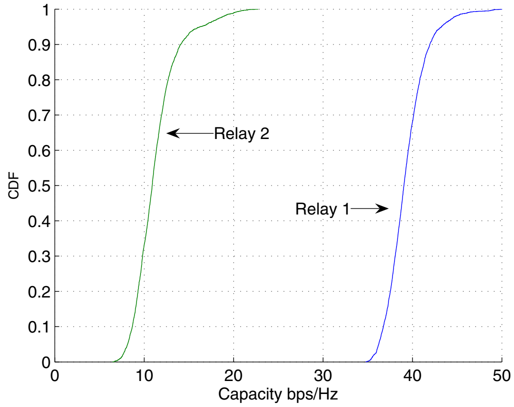

Fig. 3. The capacity of the relay channels.

B. Simulation results

To further analyze the position effect of the relay, we simulate with 2 pairs of cross-polarized patch antennas at the source, relay and the destination. We apply the widely used assumption in the simulation [3], [4], [11], [12]. The source-relay distance d_{SR} and relay-destination distance d_{RD} are normalized with respect to the source-destination distance d_{SD} when calculating the path loss. The transmit power is set as 1 Watt. The element spacing for source is 4λ, and the spacing for both the relay and the source is assumed to be 0.5λ. Intuitively, the relay should lie within the right semi circle with the source at the center, which radius is determined by d_{SD}, i.e., $0 < d_{SR} < d_{SD}$, and $-\pi/2 < \theta_S < \pi/2$. θ_S is the angle between source-relay link and source-destination link as indicated in Fig. 1. The Ricean K factor is assumed as $K = 13 - 0.03d\,(dB)$ as in [8]. Furthermore, we assume that the polarizations of all the antennas at the source, relay and destination are perfectly matched, i.e., $\theta_p = 0$.

The capacity of the relay channel with 2 pairs of cross-polarized patch antenna at the relay is depicted in polar coordinate system in Fig. 4 with $r = d_{SR}/d_{SD}$, $\theta = \theta_S$ and the source at the origin. Since the distance is normalized when calculating the path loss, the channel can always have good SNR compared with measurement in NLOS scenario. Therefore the plot only illustrates the relative capacity of the channel, not the realistic one which depends on the real propagation environment. We have a general information of the capacity of the channel with the change of the location of the relay. It is observed that the channel performs better when $-30^\circ < \theta_S < 30^\circ$ and $0.3 < d_{SR}/d_{SD} < 0.8$. This is for the reason that the destination is far enough to make this area

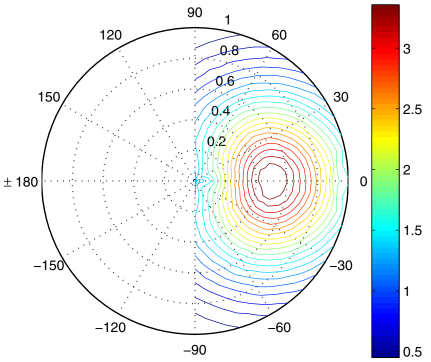

Fig. 4. The capacity of the relay channel with 2 pairs of cross-polarized patch antenna at the relay. The color bar indicates the capacity in bps/Hz

NLOS to both the source and the destination, i.e., $\mathbf{H}_{SR}$ and $\mathbf{H}_{RD}$ are able to obtain higher rank at the same time in this scenario.

The low capacity when the relay lies in the LOS scenario of the source is caused by the high correlation of the channel. Increasing the number of antennas at the relay cannot increase the capacity. In the case that d_{SR} and θ_S are both large, the relay is still NLOS to both the source and the destination, but the channel suffers more loss because of the longer propagation path length from the relay to destination, which decreases the SNR at the receiver, and hence the capacity.

The above discussion is only valid for ULA patch antennas, if omnidirectional antennas are used, the influence of θ_S is not obvious. The capacity for omnidirectional antennas at the relay is a little bit higher than that for ULA patch antennas in LOS scenario. In NLOS scenario, they perform almost the same. This comparison is based on the assumption that both kinds of antennas have the same power density. However, the power density of directional antenna is usually higher than that of omnidirectional antennas in reality.

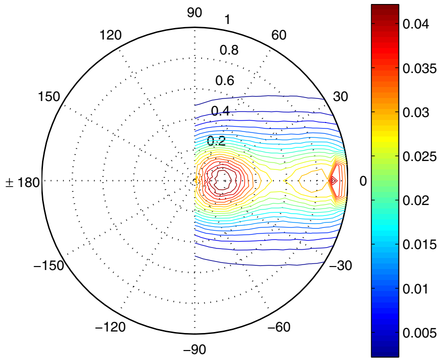

Fig. 5. The capacity of the relay channel with 2 pairs of cross-polarized patch antenna at the relay in low SNR case. The color bar indicates the capacity in bps/Hz

In the aforementioned analysis, since the source-destination distance is normalized to 1 and the noise is assumed to be 0 dBw, the SNRs at the relay and the terminal are always larger than 0 dB for source-relay and relay-destination links. Hence the relay can exploit the spatial diversity when it is placed in a rich scattering environment, and outperforms the case when it is in LOS scenario. However, in the low SNR case, the situation is different. The simulation result is plotted in Fig. 5. The distance is no longer normalized, and we assume that the SNR is around 0 dB for both source-relay link and relay-destination link when the relay is put in the central region between the source and the destination. The transmit power and noise level are then chosen accordingly. From the figure we can see that the system capacity is higher when the relay is placed near source or destination. In addition, it can be observed that the result is better if the relay is close to the source. This further validates the discussion in [13]. It reported that the source-relay link is the weakest in the system, hence it is much recommended to select relays which are in close proximity to the source in order to increase the source-relay link quality.

The capacity of the relay channel depends on the SNR of source-relay link and relay-destination link. When the SNR for both links can be guaranteed, the relay can be put in the NLOS scenario to improve the channel capacity. Otherwise, it is better to place the relay in the LOS scenario.

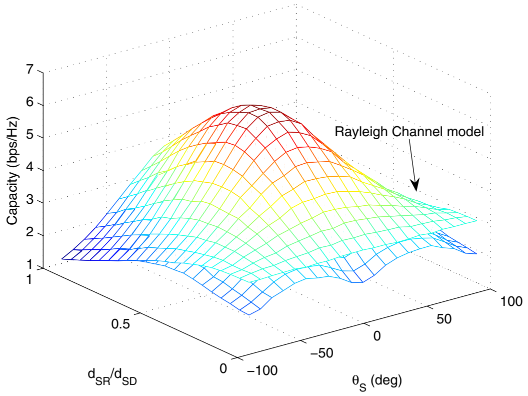

Fig. 6. The comparison between the Rayleigh channel model and our model.

Last, we compare our model with the widely used Rayleigh channel model, which assumes that the entries of both $\widetilde{\mathbf{H}}_{SR}$ and $\widetilde{\mathbf{H}}_{RD}$ are i.i.d. complex Gaussian random variables. It can be easily observed from the Fig. 6 that the Rayleigh channel model predicts much higher capacity than our model when strong LOS propagation path is present. As d_{SR} and θ_S increase, the difference between the two surfaces decreases until they overlap each other. Therefore, the Rayleigh channel model will generally overestimate the capacity of the channel if the relay does not lie in a real NLOS scenario with respect to both the source and the destination.

IV. POWER ALLOCATION

From the aforementioned analysis, we know that the capacity of the channel is relatively small when the relay is put in the LOS scenario of the source node to enlarge the coverage. In order to improve the channel performance, we use water-filling (WF) power allocation strategy at the source node to enhance more capacity. The WF-based channel can be written as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}_{RD}\mathbf{G}\mathbf{H}_{SR}\widetilde{\mathbf{x}} + \left[\begin{array}{cc} \mathbf{H}_{RD}\mathbf{G} & \mathbf{I} \end{array} \right] \left[\begin{array}{c} \mathbf{n}_{SR} \\ \mathbf{n}_{RD} \end{array} \right] \\ &= \mathbf{A}\widetilde{\mathbf{x}} + \mathbf{B} \end{aligned} \tag{7}$$

where $\widetilde{\mathbf{x}}$ consists of a power allocation matrix $\mathbf{P} = diag\{\sqrt{p_1}, \sqrt{p_2}, ...\sqrt{p_k}\}$, and a beamforming matrix $\mathbf{U} = \{u_1, u_2, ..., u_k\}$ with $\mathbf{U}\mathbf{U}^H = \mathbf{I}$. k is the number of eigen subchannels, and $\|\mathbf{P}\|_F^2 = P_0$ is the total transmitted power. The capacity of the channel is given by

$$C = 0.5\log_2\det\left(\mathbf{I} + \left(\mathbf{B}\mathbf{B}^H\right)^{-1}\mathbf{A}\widetilde{\mathbf{x}}\widetilde{\mathbf{x}}^{\mathbf{H}}\mathbf{A}^H\right). \tag{8}$$

Let us define $\mathcal{H}^H\mathcal{H} = \mathbf{A}^H\left(\mathbf{B}\mathbf{B}^H\right)^{-1}\mathbf{A}$ as the equivalent channel correlation matrix [11], and decompose the equivalent channel as

$$\mathcal{H}^H\mathcal{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H. \tag{9}$$

Hence the power allocation matrix can be determined using water-filling algorithm as $\mathbf{P} = \sqrt{[\rho\mathbf{I} - \mathbf{\Lambda}^{-1}]^+}$. The operation $[\cdot]^+$ denotes the component-wise maximum with zero, and ρ is the water-filling level chosen to satisfy $\|\mathbf{P}\|_F^2 = P_0$. The beamforming matrix $\mathbf{U}$ is obtained as in (9).

The comparison of the relay channel capacity for uniform power allocation channel and the WF-based channel is plotted in Fig. 7. It is obvious that the WF-based relay channel outperforms the uniform power allocation channel. The improvement of the capacity in LOS scenario is more than that in NLOS scenario, since WF puts more energy into the strong channel eigenmodes.

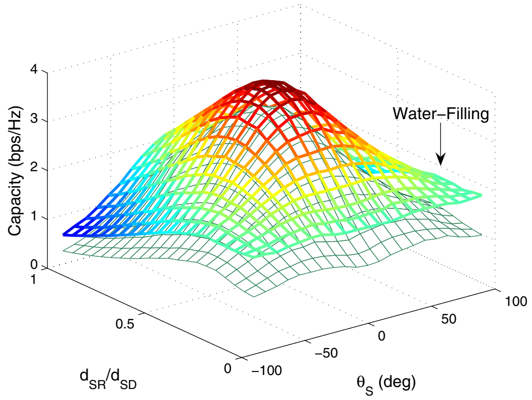

Fig. 7. The comparison of the channel capacity for uniform power allocation channel and the WF-based channel.

V. CONCLUSION

In this paper, we propose a more realistic channel model for the two-hop relay system, which takes into account the effect of the relay location and the impact of LOS. Measurement and simulation results show that the location affects both channel characteristics and performance significantly. From the simulation we know that if the destination is far enough away from the source to make their central area NLOS to both of them, it is better to place the relay in the region close to the LOS component between the source and destination. When the relay is in the LOS scenario with respect to either source or destination, the configuration of the antenna will also affect the channel capacity. Increasing the number of relay antennas has little improvement in capacity because of the high correlation of the channel. The water-filling algorithm is employed to adapt the transmission strategy according to the channel characteristics. Comparison shows that the WF-based relay channel outperforms the uniform power allocation relay channel, especially when the relay is in the LOS scenario of the source, i.e., some kind of adaptive transmission may be useful.

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